INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

MATHEMATICAL METHODS

Standard Level

Wednesday 5 May 1999 (morning)

Paper 2 | 2 hours

This examination paper consists of 2 sections, Section A and Section B.
Section A consists of 4 questions.
Section B consists of 2 questions.
The maximum mark for Section A is 80.
The maximum mark for Section B is 40.
The maximum mark for this paper is 120.

INSTRUCTIONS TO CANDIDATES

Do NOT open this examination paper until instructed to do so.

Answer all FOUR questions from Section A and one question from Section B.

Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.

EXAMINATION MATERIALS

Required:
IB Statistical Tables
Millimetre square graph paper
Calculator
Ruler and compasses

Allowed:
A simple translating dictionary for candidates not working in their own language

FORMULAE

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Arithmetic series: $S_n = \dfrac{n}{2}\{2a + (n-1)d\}$

Geometric series: $S_n = \dfrac{a(r^n - 1)}{r - 1}$, $r \neq 1$

Arc length of a circle: $s = r\theta$

Area of a sector of a circle: $A = \frac{1}{2}r^2\theta$

Area of a triangle: $A = \frac{1}{2}ab\sin C$

Statistics: If $(x_1, x_2, \ldots, x_n)$ occur with frequencies $(f_1, f_2, \ldots, f_n)$ then the mean m and standard deviation s are given by

$$m = \frac{\Sigma f_i x_i}{\Sigma f_i} \qquad s = \sqrt{\frac{\Sigma f_i (x_i - m)^2}{\Sigma f_i}}, \qquad i = 1, 2, \ldots, n$$

Newton-Raphson formula: (For finding a root of $f(x) = 0$)

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Integration by parts: (Analytical Geometry and Further Calculus Option only)

$$\int u \frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v \frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

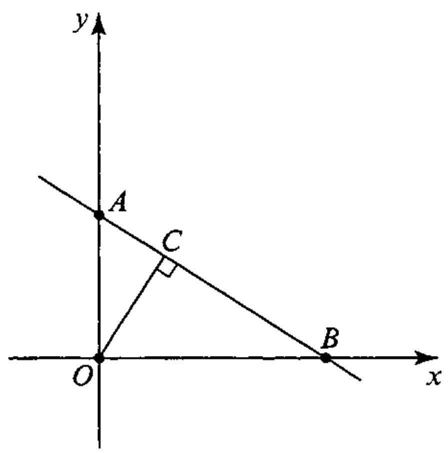
A correct answer with **no** *indication of the method used will normally receive* **no** *marks. You are therefore advised to show your working.*

SECTION A

Answer all FOUR questions from this section.

1. *[Maximum mark: 16]*

The line (AB) has equation $3x + 4y - 15 = 0$. The point O is the origin.



(a) Find

(i) the coordinates of points A and B; *[3 marks]*

(ii) the area of triangle OAB; *[2 marks]*

(iii) the length AB. *[2 marks]*

The line (OC) is perpendicular to (AB).

(b) Using the results above, or otherwise, find

(i) the length OC; *[3 marks]*

(ii) the area of triangle OCB; *[4 marks]*

(iii) the length AC. *[2 marks]*

2. *[Maximum mark: 24]*

The function f is given by

$$f(x) = \frac{2x+1}{x-3}, \quad x \in \mathbb{R}, \ x \neq 3 .$$

(a) (i) Show that $y = 2$ is an asymptote of the graph of $y = f(x)$. *[2 marks]*

(ii) Find the vertical asymptote of the graph. *[1 mark]*

(iii) Write down the coordinates of the point P at which the asymptotes intersect. *[1 mark]*

(b) Find the points of intersection of the graph and the axes. *[4 marks]*

(c) Hence sketch the graph of $y = f(x)$, showing the asymptotes by dotted lines. *[4 marks]*

(d) Show that $f'(x) = \dfrac{-7}{(x-3)^2}$ and hence find the equation of the tangent at the point S where $x = 4$. *[6 marks]*

(e) The tangent at the point T on the graph is parallel to the tangent at S.

Find the coordinates of T. *[5 marks]*

(f) Show that P is the midpoint of $[ST]$. *[1 mark]*

3. *[Maximum mark: 16]*

One thousand candidates sit an examination. The distribution of marks is shown in the following grouped frequency table.

Marks	1–10	11–20	21–30	31–40	41–50	51–60	61–70	71–80	81–90	91–100
Number of candidates	15	50	100	170	260	220	90	45	30	20

(a) **Copy** and complete the following table, which presents the above data as a cumulative frequency distribution. *[3 marks]*

Mark	$\leqslant 10$	$\leqslant 20$	$\leqslant 30$	$\leqslant 40$	$\leqslant 50$	$\leqslant 60$	$\leqslant 70$	$\leqslant 80$	$\leqslant 90$	$\leqslant 100$
Number of candidates	15	65					905			

(b) Draw a cumulative frequency graph of the distribution, using a scale of 1 cm for 100 candidates on the vertical axis and 1 cm for 10 marks on the horizontal axis. *[5 marks]*

(c) Use your graph to answer parts (i)–(iii) below.

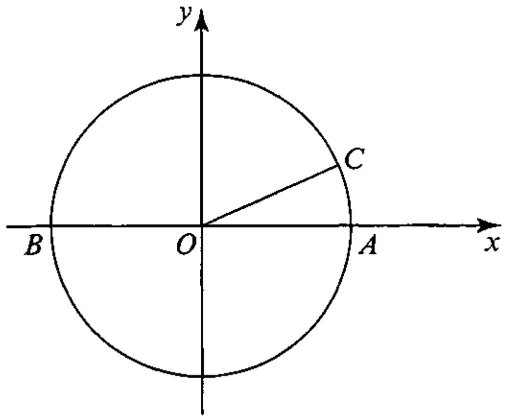
(i) Find an estimate for the median score. *[2 marks]*

(ii) Candidates who scored less than 35 were required to retake the examination. How many candidates had to retake? *[3 marks]*

(iii) The highest-scoring 15% of candidates were awarded a distinction. Find the mark above which a distinction was awarded. *[3 marks]*

4. *[Maximum mark: 24]*

(i) The circle shown has centre O and radius 6. $\overrightarrow{OA}$ is the vector $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$, $\overrightarrow{OB}$ is the vector $\begin{pmatrix} -6 \\ 0 \end{pmatrix}$ and $\overrightarrow{OC}$ is the vector $\begin{pmatrix} 5 \\ \sqrt{11} \end{pmatrix}$.



(a) Verify that A, B and C lie on the circle. *[3 marks]*

(b) Find the vector $\overrightarrow{AC}$. *[2 marks]*

(c) Using an appropriate scalar product, or otherwise, find the cosine of angle OAC. *[3 marks]*

(d) Find the area of triangle ABC, giving your answer in the form $a\sqrt{11}$, where $a \in \mathbb{N}$. *[4 marks]*

(ii) Let $M = \begin{pmatrix} a & 2 \\ 2 & -1 \end{pmatrix}$, where $a \in \mathbb{Z}$.

(a) Find M^2 in terms of a. *[4 marks]*

(b) If M^2 is equal to $\begin{pmatrix} 5 & -4 \\ -4 & 5 \end{pmatrix}$, find the value of a. *[2 marks]*

(c) Using this value of a, find M^{-1} and **hence** solve the system of equations:

$$-x + 2y = -3$$
$$2x - y = 3$$

[6 marks]

SECTION B

Answer ONE question from this section.

Analytical Geometry and Further Calculus

5. *[Maximum mark: 40]*

(i) The function f is such that $f''(x) = 2x - 2$.

When the graph of f is drawn, it has a minimum point at $(3, -7)$.

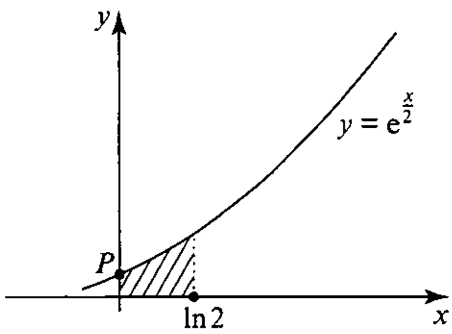
(a) Show that $f'(x) = x^2 - 2x - 3$ and hence find $f(x)$. *[6 marks]*

(b) Find $f(0)$, $f(-1)$ and $f'(-1)$. *[3 marks]*

(c) Hence sketch the graph of f, labelling it with the information obtained in part (b). *[4 marks]*

(**Note**: It is **not** necessary to find the coordinates of the points where the graph cuts the x-axis.)

(ii) The diagram shows part of the graph of $y = e^{\frac{x}{2}}$.



(a) Find the coordinates of the point P, where the graph meets the y-axis. *[2 marks]*

The shaded region between the graph and the x-axis, bounded by $x = 0$ and $x = \ln 2$, is rotated through 360° about the x-axis.

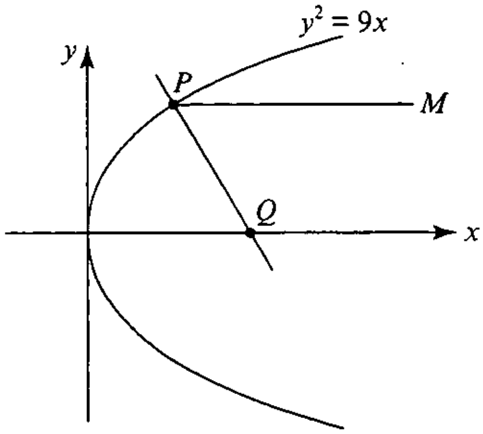
(b) Write down an integral which represents the volume of the solid obtained. *[4 marks]*

(c) Show that this volume is π. *[5 marks]*

(*This question continues on the following page*)

(Question 5 continued)

(iii) The parabola shown has equation $y^2 = 9x$.



(a) Verify that the point P (4 , 6) is on the parabola. *[2 marks]*

The line (PQ) is the normal to the parabola at the point P, and cuts the x-axis at Q.

(b) (i) Find the equation of (PQ) in the form $ax + by + c = 0$. *[5 marks]*

(ii) Find the coordinates of Q. *[2 marks]*

S is the point $\left(\frac{9}{4}, 0\right)$.

(c) Verify that $SP = SQ$. *[4 marks]*

(d) The line (PM) is parallel to the x-axis. From part (c), explain why (QP) bisects the angle SPM. *[3 marks]*

Further Probability and Statistics

6. *[Maximum mark: 40]*

(i) A box contains 35 red discs and 5 black discs. A disc is selected at random and its colour noted. The disc is then replaced in the box.

(a) In eight such selections, what is the probability that a black disc is selected

(i) exactly once? *[3 marks]*

(ii) at least once? *[3 marks]*

(b) The process of selecting and replacing is carried out 400 times.

(i) What is the expected number of black discs that would be drawn? *[2 marks]*

(ii) Use a normal approximation to the binomial distribution to estimate the probability that a black disc is selected

(a) at least 48 times; *[5 marks]*

(b) exactly 48 times. *[5 marks]*

(ii) A continuous random variable X has the probability density function

$$f(x) = kx, \text{ for } 0 \leqslant x \leqslant 5 ;$$
$$= 0 \text{ , elsewhere .}$$

Find the value of

(a) k ; *[4 marks]*

(b) $\mathrm{E}(X)$; *[4 marks]*

(c) the variance; *[4 marks]*

(d) $p(2 \leqslant X \leqslant 3)$; *[4 marks]*

(e) the median of X . *[6 marks]*
